

Eliminating the Free Radiation Field from Electrodynamics

Reinhold Brückner
(*1963, Konstanz)

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We never see "Light" - we are always looking at something

Both in the classical and wave-mechanical Lagrangean treatment of electrodynamics, interaction of charged matter with an electromagnetic field comes about through a term

$$(1) \quad A_\mu j^\mu$$

where A is the 4-vectorpotential and j are the particle current densities.

The system is still open since the 4-vectorpotential A needs to be defined.

One usually treats A as a new field with its own, free Lagrangean.

However, this is redundant since A can be constructed from charged sources through a Green's function

$$(2) \quad A_\mu(\vec{r}, t) = \int dV' dt' G(\vec{r}, \vec{r}', t, t')_{\mu\nu} J_\nu(\vec{r}', t')$$

It necessitates fixing the gauge which is acceptable since the resultant observables are the same. This can be inserted into the Lagrangean

$$(3) \quad \frac{1}{2} \int dV dt A_\mu(\vec{r}, t) j^\mu(\vec{r}, t) = \frac{1}{2} \int dV dt \int dV' dt' j_\mu(\vec{r}, t) G(\vec{r}, \vec{r}', t, t')_{\mu\nu} j_\nu(\vec{r}', t')$$

The factor 1/2 compensates double-counting during the variation procedure.

This very form reduces us to Lorentz gauge which is okay since freedom of gauge is a mathematical artefact.

A free electrodynamic Lagrangean becomes redundant since it does not connect to charged fields anymore.

The Green function formulation supposes that the currents do not contain the A -fields implicitly.

This is true for classical mechanics where currents only contain observables (charge density, velocity).

For wave mechanics, the only known wave equation to not include the A -field in the currents is the Dirac equation. It is the most fundamental one available and maybe suited for that task.

One might ask whether we do not insert the Green function with hindsight as the outcome of the "usual" procedure. Absolutely! All Lagrange formalism is hindsight. First were always the dynamical equations, be that Newton's or Maxwell's, then came the Lagrangean which was crafted to reproduce said dynamical equations.

From Maxwell's equations, we derive vector potentials first and Green functions after.

It is only then, that we look at a Lagrangean formulation for the matter field and the matter field only since electromagnetic fields are defined through the action on matter.

The extension to a "free" electromagnetic field without any charges whatsoever is, for the time being, an unjustified idealisation.

We usually go around this by supposing "free" fields to be generated by sources "far away" and confound it with a "source-free" field. The fields very much resemble "true" free fields and give successful descriptions.

Aren't we then just being picky?

No, since our reformulated interaction term touches upon the thorny question of electromagnetic energy and momentum (Poynting vector) *densities*.

The Lagrangean exhibits the usual invariances (displacement, rotation etc.) and we expect to get a number of conserved quantities. Some overall, some hopefully local for which one can formulate continuity equations and hence "densities".

All of these densities will now be *matter-bound*. There will be no densities in empty space.

This seems odd at first glance since we got so used to radiation "density".

There are, however, situations where this empty-space quantity shows its feet of clay.

For example, the Poynting vector as defined by $\vec{P} = \vec{E} \times \vec{B}$ allows the nonsensical situation of crossed, *static*, magnetic and electric fields (a coil circling a capacitor) giving a "formal" finite Poynting vector although there is manifestly no flow of momentum.

The reason, of course, is the casual notion of "energy density". The very derivation of "field momentum" through the force action on charges is faulty, because it does not distinguish between external fields and fields proper to the matter involved.

How about the actual meaning of radiation densities? It is the presence of forces emanating from some sources. These forces can move matter and as they do, the moved matter will emit forces, that act back on the sources. We would not have an issue with that if the speed of light were infinite since this means action at a distance. Well, this becomes action at a distance with a delay. (This will give an issue with Noether's theorem which relies on a single time for all fields.)

We can even go so far as to call the notion of a "field" faulty from the start.

Where does it come from?

We write the force action on some body as $\mathbf{F} = e\mathbf{E}$ where (e) is the charge of some "probe".

Except that there are no "probes". Best illustration is image charges: an uncharged conductor has no field. Yet, a charge "probe" becomes attracted to it.

It becomes clear that there is (so far) no field but *force actions between charges*.

Are self-fields possible?

It depends on what kind of charge densities you have in mind. For point-like classical charges, there will be no room for self-interaction, hence no radiation reaction.

For continuous charge densities, sure! There is no short-range energy divergence: Green function rises as $1/R$ and phase space behaves as dRR^2 .

A-field $1/R$ long-range behaviour will attribute a much larger role to surrounding matter.

"Radiation" in a system of "lonely" charges will not occur since field momentum is bound to these charges. Only when other charges, even far away, are present will "radiation" loss happen.

The actual mechanism of radiation is still in the dark but we can ascertain that we will need polarizable or movable matter as receivers.

An array of fixed charges insensitive to external fields will not do.

Light only shines when someone can see it.

Last not least, let us have a closer look at the aforementioned "conserved quantities".
 We limit ourselves to classical point-like particles.
 For momentum conservation (one particle i), we consider the Hamilton canonical momentum

$$(4) \quad \vec{P}_i(\vec{r}, t) = \gamma(\vec{v}_i(t)) m_i \vec{v}_i(\vec{r}_i(t)) + q_i \vec{A}(\vec{r}_i, t)$$

The A-term can be a "reservoir" of primitive momentum during delay of transmission between charges apart from being a quantity of its own. For charges far apart, the A-term vanishes and we recover primitive linear momentum.

The "surface term" of momentum change reduces to $q_i \nabla A_0(\vec{r}_i, t)$. That is the best that we can get. For action at a distance (c infinite), the gradient terms cancel overall.

With a delay (c finite), that is not the case anymore.

We would like to derive energy "conservation" by the usual procedure, namely forming

$$(5) \quad \vec{v}_i \frac{d}{dt} \vec{P}_i(r_i(t), t) = q_i \langle v_i | \nabla \rangle A_0(\vec{r}_i, t) = q_i \frac{d}{dt} A_0(\vec{r}_i, t) - q_i \frac{\partial}{\partial t} A_0(\vec{r}_i, t)$$

The Lorentz gauge condition

$$(6) \quad \frac{\partial}{\partial t} A_0(\vec{r}_i, t) = -\langle \nabla | A \rangle$$

gives

$$(7) \quad \vec{v}_i \frac{d}{dt} \vec{P}_i(r_i(t), t) = \frac{d}{dt} q_i A_0(\vec{r}_i, t) + \langle \nabla | q_i A \rangle$$

ignoring the "surface" term, the energy becomes

$$(8) \quad W(\vec{r}, t) = q_i A_0(\vec{r}_i, t)$$

This is a surprise! But think a second: kinetic energy in the absence of interaction is a constant. All change is contained in the interacting potential which becomes the actual energy.

Vectorpotential contributes a "surface" term which becomes a matter-bound, *energy flow*.

Although simple, these quantities are non-local and (non-conserved) energy and momentum content become some kind of a mysterious fait accompli.

Once again, at finite c, *overall* energy will *not* be "perfectly" conserved.

Before, we had put the "missing" parts (of energy etc.) into a "free field" but we do not have this leisure anymore.

Energy and momentum are conserved only *statistically*.

Quantum Randomness becomes Classical Randomness